On mathematical digital competency for teaching: The case of an expert teacher

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In this paper, we present a mathematics teacher's reflections on the design and experimentation of an activity sequence involving transitions from 'pen-and-paper' mathematical explorations to mathematical explorations within three different digital environments, GeoGebra, the Scratch programming environment and Excel. We look at her arguments for supporting students' development of Mathematical Digital Competency (MDC) and reflect on her instrumental orchestration approaches. We then argue and discuss the idea of MDC for teaching (MDCT) using this expert teacher's case as an exemplar for such practice.

Keywords: Digital competencies, instrumental orchestration, mathematical competencies, mathematical digital competency, mathematics teachers.

Introduction

In a recent paper, Geraniou and Jankvist (2019) argue that for mathematics students of today, their understanding of mathematical concepts involved in several mathematical situations might be "almost inseparable from the digital tools and the students' instrumented techniques" usually associated with those situations (p. 43). Hence, "for such students, it is no longer only about either mathematical competency or digital competency. It becomes about mathematical digital competency" (p. 43). On this basis, they provide a first attempt at a definition of such MDC (see the following section). Accepting that mathematical digital competency (MDC) thus is an important component for students in 21st century mathematics education, it is obvious to ask about MDC for teachers. Geraniou and Jankvist (2020) name this mathematical digital competency for teaching (MDCT) and provide a discussion of which potential theoretical frameworks might function—or network—well with the notion of MDC. These include (the theory of) instrumental orchestration (TIO), the Danish KOM framework's six didactico-pedagogical competencies of mathematics teachers, mathematical knowledge for teaching (MKT), and the associated so-called TPACK (technological pedagogical content knowledge) framework. With reference to Geraniou and Jankvist (2019), Tabach (2021) picked up from a TPACK, and thus MKT, point of view to conclude:

> Returning to the issue of teachers' digital mathematical competencies with which I opened the talk, I believe that the MDC defined by Geraniou and Jankvist (2019) also applies to teachers. Beyond this is a complementary set of competencies, specifically didactical digital mathematical competencies, that are relevant to the work of mathematics teachers. In this talk I hinted at some of these, which I believe constitute a fruitful field for future research (Tabach, 2021, p. 44).

Tabach's "didactical digital mathematical competencies" correspond to our notion of MDCT. In this paper, we address this "complementary set of competencies" by taking a more empirical look at what MDCT might look like when practiced in a classroom by providing an illustrative case of an expert mathematics teacher in programming, Grace. The case stems from a larger project related to students' computational thinking (CT) and MDC and data was collected by the third author. Based on the theoretical basis of MDC and the empirical case, we attempt answers to the following exploratory research question: *Which components should MDCT at least encompass?* Before engaging into the empirical case, sharing more information about Grace, and the educational setting surrounding this, we provide a thorough description of the theoretical constructs on which we will rely: MDC and TIO.

Mathematical Digital Competency and Teacher Competencies

In our past work, we argued that when students interact with a piece of software in their efforts to solve a mathematical task, their digital competencies and their mathematical competencies are enacted and intertwined (Geraniou & Jankvist, 2019). Building upon the Danish mathematics competencies framework, KOM (Niss & Højgaard, 2019), and combining the Theory of Instrumental Genesis (Trouche, 2005) and Vergnaud's (2009) Theory of Conceptual Fields, Geraniou and Jankvist (2019) advanced the theoretical construct of students' MDC, proposing that students possessing such display the following characteristics:

- "[MDC1]: *Being able to engage in a techno-mathematical discourse*. In particular, this involves aspects of the artefact-instrument duality in the sense that instrumentation has taken place and thereby initiated the process of becoming techno-mathematically fluent.
- [MDC2]: *Being aware of which digital tools to apply within different mathematical situations and context, and being aware of the different tools' capabilities and limitations*. In particular, this involves aspects of the instrumentation–instrumentalisation duality.
- [MDC3]: *Being able to use digital technology reflectively in problem solving and when learning mathematics*. This involves being aware and taking advantage of digital tools serving both pragmatic and epistemic purposes, and in particular, aspects of the scheme-technique duality, both in relation to one's predicative and operative form of knowledge" (p. 43).

For teachers to assist students in developing their MDC, besides possessing MDC to some extent themselves, they need MDC for teaching (MDCT). Niss and Højgaard's (2019) definition of mathematical competence as "someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations" (p. 12) should be taken into account when considering MDC for teaching. Teacher competencies are not to be mistaken for solely a set of traits or skills; rather, they are defined in the way in which specific actions are implemented and the intentionality and importance that both precede and follow those actions (Winch, 2017). Teacher competencies are defined as the personal qualities—specifically, knowledge, beliefs, and motivation, as distinguished from behaviours and interactions—required for teachers to meet the demands in their profession (Fauth et al., 2019). Krumsvik and Jones's (2013) characterisation of teacher's digital competencies involves two dimensions, that of the competency to use technology for personal use and that of the competency to use technology in pedagogical settings. This has also been conceptualised as the double instrumental genesis (Haspekian, 2011), a process involving a pedagogical instrumental genesis on top of a teacher's personal instrumental genesis.

Theory of Instrumental Orchestration (TIO)

To analyse how the enactment of the double instrumental genesis takes place, and in fact how a teacher manages and orchestrates the use of digital technology in mathematical learning situations, we use TIO. TIO was derived by Trouche (2004) and later elaborated by Drijvers et al. (2014) as "the teacher's intentional and systematic organisation and use of the various artefacts available in a learning environment—in this case a computerised environment—in a given mathematical task situation, in order to guide students' instrumental genesis" (p. 191). TIO involves the following three elements: (a) a *didactic configuration*, that is the arrangement of artefacts in the teaching environment; (b) an *exploitation mode*, or in other words the approach a teacher chooses to exploit a didactical configuration to assist their didactical intentions; (c) a *didactical performance*, that entails the decisions a teacher needs to make instantly, while teaching to accommodate the chosen didactic configuration and exploitation mode. Seven orchestrations have been identified for whole class teaching in up-to-date research studies and one for students working alone or in pairs with technology (Drijvers et al., 2014): (1) *technical-demo* orchestration concerns demonstration of tool techniques by the teacher; (2) *link-screen-board* orchestration, where the teacher stresses the relationship between what happens in the technological environment, and its representation in the conventional mathematics of paper, book and board; (3) *discuss-the-screen* orchestration concerns a whole-class discussion about what happens on the computer screen; (4) *explain-the-screen* orchestration concerns whole-class explanation by the teacher, guided by what happens on the computer screen; (5) *spot-and-show* orchestration, where students' reasoning is brought to the fore through the identification of their work during the preparation of the lesson and its use in a classroom discussion; (6) S*herpa-at-work* orchestration, a so-called Sherpa student (Trouche, 2004, 2005) uses the technology to present his or her work, or carry out actions on the teacher's request; and (7) *work-and-walk-by* orchestration, which is where the didactical configuration and the corresponding resources basically consist of the students sitting at their technological devices, and the teacher walking around in the classroom. All these seven orchestrations involve whole-class teaching (Drijvers et al., 2014), and have been derived to describe the teacher's role in supporting and guiding students while they interact with a digital resource, as well as helping them learn the mathematics involved and how to use the resource.

The case of expert teacher Grace

The empirical basis of this paper relies on the collaborative work between the third author and Grace. Grace is a mathematics teacher with 37 years of experience, a mathematics advisor in her municipality, and current member of the mathematics expert group for the Danish Ministry of Education. Moreover, she has a particular expertise and interest in programming, leading a non-profit organization that involves children in coding for seven years. The collaborative work began by offering Grace a didactical sequence, where students should combine their mathematical and programming learning to solve a task. In particular, the goal of the offered task was to code a program in Scratch that draws a regular polygon of any given number of sides (see Figure 1). The original design is inspired by Papert's (1980) Turtle geometry, and the decisions on the order of coding different polygons were informed by the *ScratchMaths* project (Benton et al., 2017). Data were collected in one pre- and one post-intervention interviews with Grace, and video and audio recordings from the classroom experience and were transcribed, anonymised and translated. The researchers' reflections presented below are based on all these data. The implemented version of the task consisted of three 90-minute sessions with one of Grace's 6th-grade classes, who were introduced to Scratch in the first session. These sessions are summarised below.

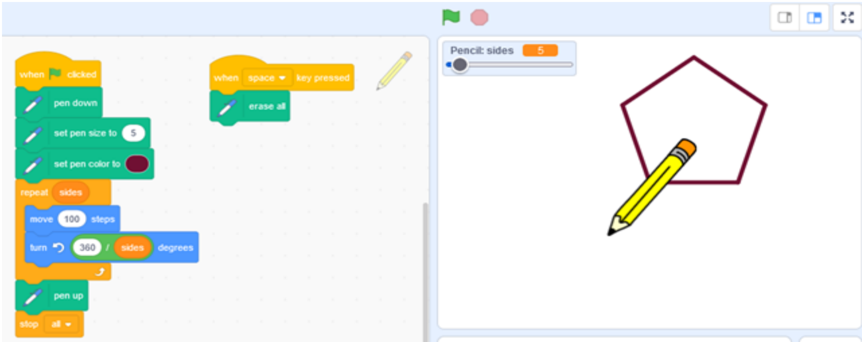

Figure 1: A sample solution of the original proposed task

Session 1: Introduction to Scratch's pen environment. Grace invited students to open Scratch and explore its capabilities. Every so often, students would share with the class what they have found. Grace steered the conversation toward key features: create and remix blocks, the green flag and the pen environment. The session ended with some pre-made code that students should fix.

Researchers' reflections on Session 1. Grace had already reflected on the best tools to use to teach the mathematical topic of 'Angles in a polygon' (MDC2) and aimed for the students to become familiarised with the Scratch environment and its coding language. Grace used a combination of orchestrations, such as: *work-and-walk-by* to support students when and if needed while they interacted with Scratch in their allocated computers; *technical-demo*, *discuss-the-screen* and *explain-the screen* in an effort to draw students' attention to key features mentioned above (e.g., create and remix blocks), and allow students to learn and appreciate what all these key features in Scratch do. Such an approach prepared students to interact with Scratch and initiated students' *engagement in a techno-mathematical discourse* (MDC1), as well as their awareness of Scratch's *capabilities and limitations* (MDC2). At the end of the session, Grace presented students with a pre-made Scratch code and asked students to correct it. In this activity, students began to consider Scratch as an instrument to support them in their mathematical explorations and therefore continued to develop their techno-mathematical discourse. This could not have taken place without Grace's support and guidance. She used several orchestrations and showcased her ability in didactically configuring the activity sequence so that students began to engage with MDC1 and in particular MDC2.

Session 2: Coding polygons. After a briefing on regular polygons, Grace asked the students to code regular polygons in Scratch. The students had the freedom to choose colours, size and order. Upon sharing their findings, Grace displayed an Excel spreadsheet, where the students in collaboration should fill in the turning angle and the sum of exterior angles for each polygon. Grace's past experiences with this class of 6th graders involved training to use Excel to record and discuss tabular data (e.g., daily numbers of Covid-19 infections), which led to the use of Excel as an alternative to the blackboard. Later, she showed the students how to use another digital resource, GeoGebra, and use the "Regular Polygons" feature and record the interior angles of each polygon (starting from a triangle) and the sum of interior angles in the same Excel spreadsheet (see Figure 2).

	A	B	C	D	E	F	G
1	Kant	Vinkel Scratch	Vinkelsum			Vinkel GeoGebra	Vinkelsum
2							
3	3-kant	120	360			60	180
4	4-kant	90	360			90	360
5	5-kant	72	360			108	540
6	6-kant	60	360			120	720
7	7-kant	51	360			128,57	899
8	8-kant	45	360			135	1080
9	9-kant	40	360			140	1260
10	10-kant	36	360			144	1440

Figure 2: Excel screen capture of students' collection of angles and sum of angles by means of Scratch and GeoGebra ('Kant' is 'side'; 'Vinkel' is 'angle'; 'Vinkelsum' is 'sum of angles').

Researchers' reflections on Session 2. Grace wanted to compare different approaches to creating regular polygons and investigating their interior and exterior angles and the sum of those angles. She demonstrated *awareness of which digital tools to apply within different mathematical situations and context* (in this case the focus being on either exterior angles of polygons, leading to the use of Scratch, or interior angles of polygons, leading to the use of GeoGebra) (MDC2). She drew students' attention to how the sprite in Scratch 'forced' students to visualise the direction the sprite was going to move; hence, recognise that the focus was indeed on identifying how many degrees the sprite had to 'turn' to draw the next side of the polygon, and that 'turn' was in fact the exterior angle of the polygon. She also drew students' attention to the angle indicated in their GeoGebra constructed polygons, which indeed was the interior angle of those polygons. She used the *explain-the-screen* orchestration to discuss the two different computations taking place in Scratch and GeoGebra, but also to showcase the data on angles of polygons presented in a different digital resource, Excel. This latter action encouraged students to reflect on and compare exterior and interior angles of polygons of different number of sides, and spot any patterns, e.g., the sum of exterior angles of any polygon always being 360 degrees. We can argue that she took the *link-screen-board* orchestration a step further and instead of using the *physical* board to link what was happening in Scratch and in GeoGebra, she decided to use a 3rd digital resource, Excel, that allowed her instantly to present the sum of angles in a polygon in a tabular representation. In a way, she used a *link-different-digital-resources* orchestration. She also *took advantage* of the three digital resources *serving both epistemic and pragmatic purposes* for her own teaching and her students' mathematical learning (MDC3). All her decisions reveal her possession of all three elements of MDC and her awareness and application of didactical pedagogical strategies for teaching mathematics with the chosen three digital resources, which can be characterised as MDCT.

Session 3: Drawing skylines. The session began with summarizing findings from the previous session 2, discussing the patterns between angles and sum of angles in both Scratch and GeoGebra, displayed in the Excel spreadsheet. Students were then encouraged to find skylines of buildings of their interest, draw them on paper, and make notes on how to code them in Scratch. Afterwards, they used Scratch to create their chosen skyline, applying their knowledge of polygons.

Researchers' reflections on Session 3. This session was dedicated to recapping and reflecting on what took place in the previous two sessions: students' development of a techno-mathematical discourse regarding the three digital resources used (MDC1); the expected gained mathematical knowledge and knowledge of how to interact with Scratch, GeoGebra and Excel, their capabilities and limitations (MDC2); the use of Scratch, GeoGebra and Excel reflectively to learn about the interior and exterior angles of polygons (MDC3). This was achieved by Grace using orchestrations such as *discuss-the-screen* and *explain-the-screen*, to draw students' attention to their past work on Scratch, GeoGebra and Excel, as well as the orchestration we proposed earlier on, *link-different-digital-resources*, which allowed Grace to move between the three different digital resources and the three different interfaces showing their mathematical work. The students and teachers' MDC were enacted once again with the last task, which was to model a skyline of a building of their choice using Scratch and thus allowed for consolidating their gained mathematical knowledge on polygons and techno-mathematical discourse (MDC1). This last teaching session actually engaged students the most, as they used a real-life context of their own choice and applied their MDC to produce their own codes in Scratch, leading to the creation of amazing buildings' skyline models (see Figure 3).

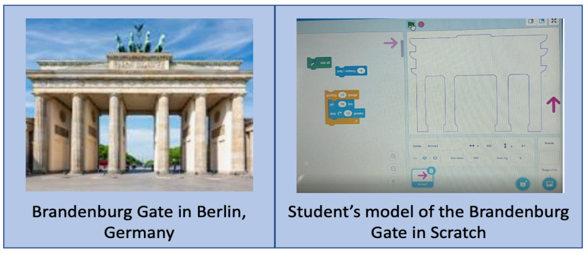

Figure 3: A student's model of the Brandenburg Gate in Berlin, Germany, as modelled in Scratch

Grace's reflections. At the end of the activity sequence, Grace was interviewed by the third author and discussed her recollections of her decisions on how best to deliver the suggested activity sequence and accommodate the transitions from pen and paper to the three digital resources used. First, Grace wanted her students to be the ones posing the problem, and exploring their own solution strategies as sub-problems appear. The context of the last task was agreed to be the drawing of skylines of buildings of their choice, by learning first to draw polygons on paper, in Scratch and in GeoGebra. Second, the solution to the problem should involve both computational and mathematical knowledge and skills. This criterion validates the task's original purpose. Third, Grace suggested involving more digital resources in their work. Based on her own trials with other classrooms, she decided to include Excel to systematize the collection of data and aid pattern recognition. She was aware of the benefits of using Excel, as it allowed seeing what the turning angle needed to create each regular polygon (triangle, square, pentagon…) is in Scratch and the interior angles in GeoGebra, in relation to the sum of angles. During the interview, Grace remembered that during Session 2, students asked "why can we not simply use GeoGebra, which draws regular polygons automatically?". She argued that she used Excel as an additional tool to support students' recollection of the different angles in polygons and enable them to compare, reflect and derive mathematical statements regarding interior and exterior angles of polygons. Scratch, GeoGebra and Excel surely played different roles in the activity sequence, and students explored their affordances and limitations, an important mathematical learning process with digital technologies and an important element of acquisition of MDC.

Conclusion

The above discussions of Grace's teaching show that she possessed MDC herself, while making didactical decisions on how the activity sequence should be exploited with students, and in particular which digital resources are the best to achieve the learning goals and why, and which instrumental orchestrations should be implemented in her teaching practice to support these goals. Her pedagogical considerations were evident when: (a) 'making' the technology accessible to students by allowing students to explore Scratch and 'debug' a code, for example, and supporting them in developing a techno-mathematical discourse (MDC1); (b) identifying the best tools to focus on exterior angles (Scratch), on interior angles (GeoGebra) and on deriving mathematical statements about interior and exterior angles as well as the sum of those angles (Excel), based on considerations of those three tools' capabilities and limitations (MDC2); (c) encouraging students to use Scratch to solve the problem of modelling the skylines of their chosen buildings and in the process apply their gained knowledge on interior and exterior angles of polygons (MDC3). Considering our exploratory research question: "Which components should MDCT at least encompass?", we draw on Niss and Højgaard's (2019) definition of mathematical competencies and based on the empirical data from Grace's example, we understand MDCT as the competencies teachers need (or have) to select and implement technology in their practice in pedagogically productive ways. Inspired and informed by the previous literature and research on students' MDC (Geraniou & Jankvist, 2019) and Niss and Højgaard's (2019) description of both students and teachers' competencies, we use the definition for students' MDC to conceptualise teachers' competencies in using technology, re-defined to suit teachers by including pedagogic elements. Therefore, we propose the following MDCT:

- [MDCT1]: Being able to engage in a techno-mathematical discourse **at a meta-pedagogic level**.
- [MDCT2]: Being aware of which digital tools to apply within different mathematical situations and context, and being aware of the different tools' capabilities and limitations, **so as to think, and act, pedagogically with these tools,** while considering the benefits and limitations of these.
- [MDCT3]: Being able to use digital technology reflectively in problem solving and when doing (learning **or teaching**) mathematics.

Our future work entails further research to investigate, validate and refine the above 'tentative' MDCT and show their importance in the effective use of digital resources in mathematics education.

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